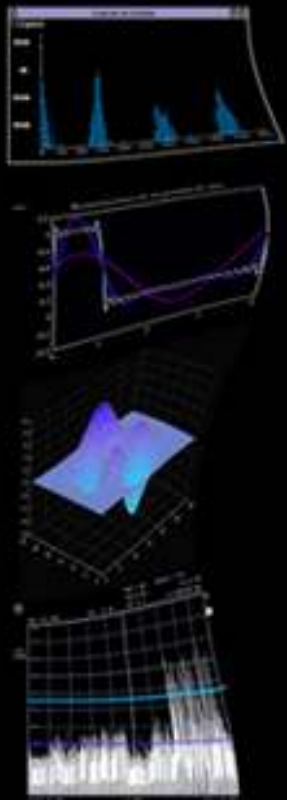
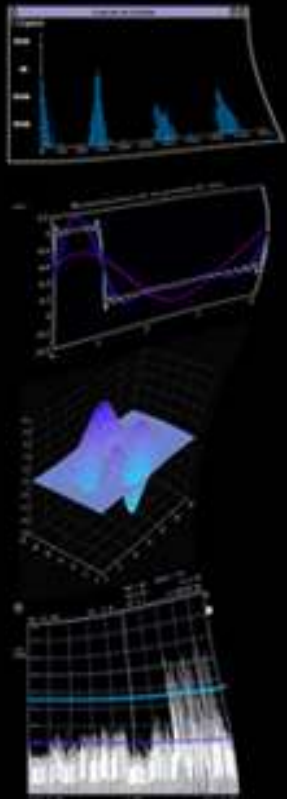


Transformada Z



Transformada Z

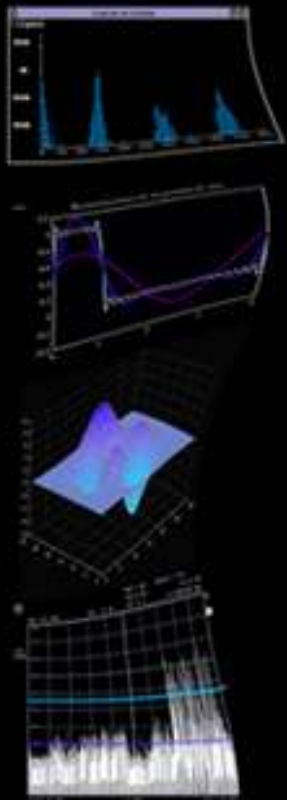
Transformada de Laplace



Transformada	Antitransformada
$TL\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt$	$TL^{-1}\{X(s)\} = x(t) = \int_{-\infty}^{\infty} X(s)e^{ts} ds$

Transformada Z

Transformada Z



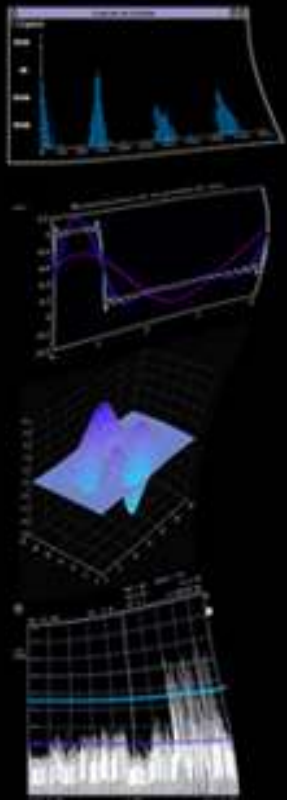
Transformada	Antitransformada
$\mathcal{TZ}\{x[n]\} = X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$	$\mathcal{TZ}^{-1}\{X(z)\} = x[n] = \frac{1}{j2\pi} \oint_C X(z)z^{n-1}dz$

Región de convergencia

$$z \in ROC \Leftrightarrow X(z) = \left| \sum_{n=-\infty}^{\infty} x[n]z^{-n} \right| < \infty$$

Transformada Z

Polos y ceros



Función polinómica

$$H(z) = \sum_{k=0}^M b_k z^{-k} = b_0 + b_1 z^{-1} + \dots + b_M z^{-M} = \frac{b_0 z^M + \dots + b_{M-1} z^1 + b_M}{z^M}$$

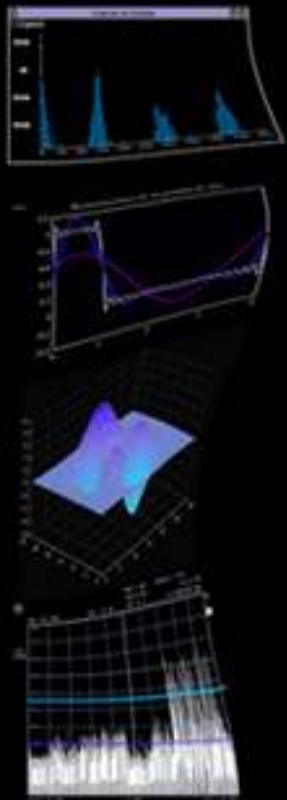
$$H(z) = z^{-M} (z - c_1) \dots (z - c_M) = z^{-M} \prod_{k=1}^M (z - c_k)$$

Transformada Z

Polos y ceros

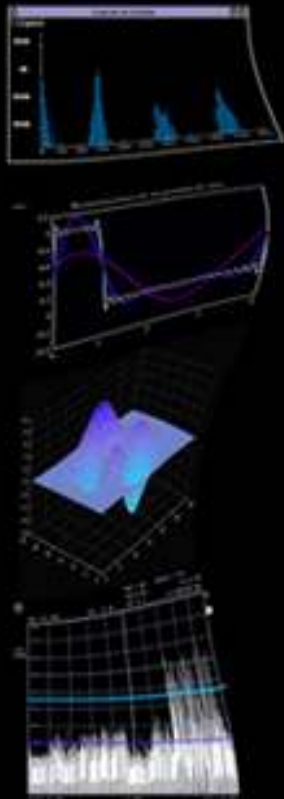
Función polinómica racional

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}}$$



Transformada Z

Polos y ceros

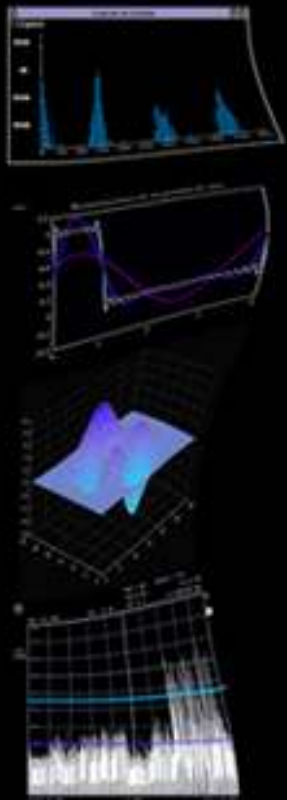


Función polinómica racional

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}} = \frac{z^N}{z^M} \frac{b_0 z^M + \dots + b_{M-1} z^1 + b_M}{a_0 z^N + \dots + a_{N-1} z^1 + a_N}$$

Transformada Z

Polos y ceros



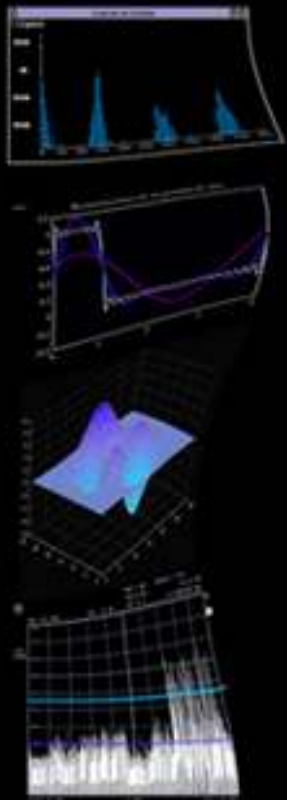
Función polinómica racional

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}} = \frac{z^N}{z^M} \frac{b_0 z^M + \dots + b_{M-1} z^1 + b_M}{a_0 z^N + \dots + a_{N-1} z^1 + a_N}$$

$$H(z) = z^{N-M} \frac{b_0 (z - c_1) \dots (z - c_M)}{a_0 (z - p_1) \dots (z - p_N)} = z^{N-M} \frac{b_0 \prod_{k=1}^M (z - c_k)}{a_0 \prod_{k=1}^N (z - p_k)}$$

Transformada Z

Polos y ceros



Función polinómica racional

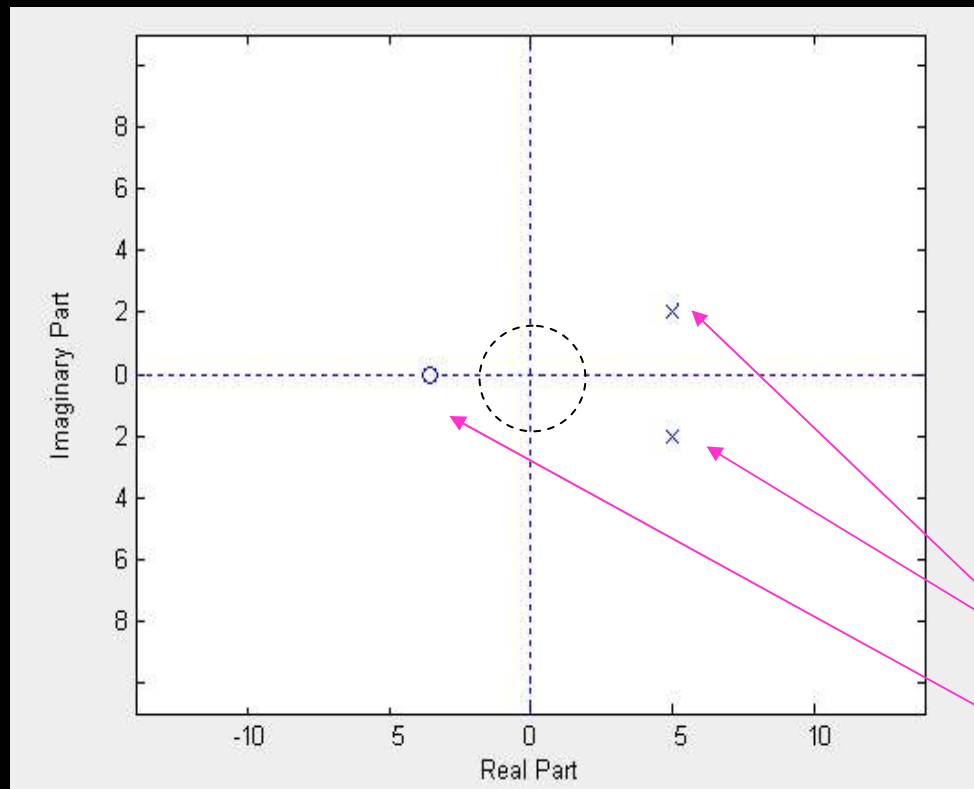
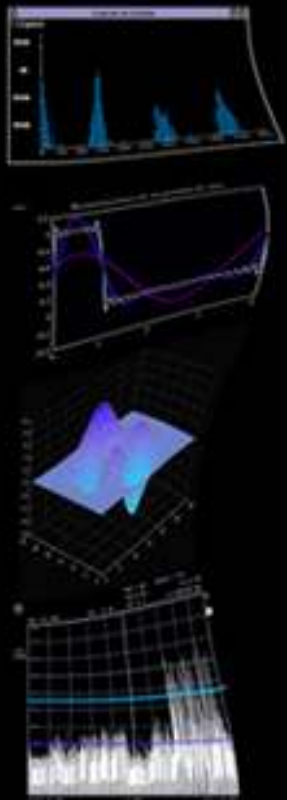
$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}} = \frac{z^N}{z^M} \frac{b_0 z^M + \dots + b_{M-1} z^1 + b_M}{a_0 z^N + \dots + a_{N-1} z^1 + a_N}$$

$$H(z) = z^{N-M} \frac{b_0 (z - c_1) \dots (z - c_M)}{a_0 (z - p_1) \dots (z - p_N)} = z^{N-M} \frac{b_0 \prod_{k=1}^M (z - c_k)}{a_0 \prod_{k=1}^N (z - p_k)}$$

$$H(z) = \frac{b_0 (1 - c_1 z^{-1}) \dots (1 - c_M z^{-1})}{a_0 (1 - p_1 z^{-1}) \dots (1 - p_N z^{-1})} = \frac{b_0 \prod_{k=1}^M (1 - c_k z^{-1})}{a_0 \prod_{k=1}^N (1 - p_k z^{-1})}$$

Transformada Z

Polos y ceros

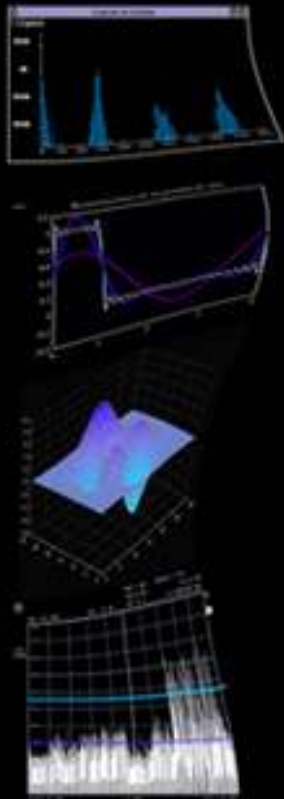


$$\lim_{z \rightarrow p_k} H(z) = \infty$$

$$\lim_{z \rightarrow z_k} H(z) = 0$$

Transformada Z

Polos y ceros



Función racional

Número de polos = número de ceros

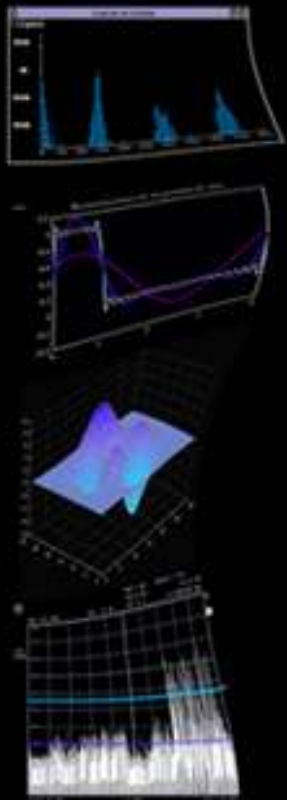
Polos y ceros **triviales**: Ubicados en $z=0$ ó $z=\infty$

Número de ceros **no triviales**: M

Número de polos **no triviales**: N

Transformada Z

Tabla de pares de funciones y transformadas



Antitransformada	Transformada	ROC
$\delta[n]$	1	z
$u[n]$	$\frac{1}{1-z^{-1}}$	$ z > 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$ z > a $
$\cos[\omega n] u[n]$	$\frac{1-z^{-1} \cos \omega}{1-2z^{-1} \cos \omega + z^{-2}}$	$ z > 1$
$\text{sen}[\omega n] u[n]$	$\frac{z^{-1} \text{sen} \omega}{1-2z^{-1} \cos \omega + z^{-2}}$	$ z > 1$
$a^n \cos[\omega n] u[n]$	$\frac{az^{-1} \text{sen} \omega}{1-2az^{-1} \cos \omega + a^2 z^{-2}}$	$ z > a $
$a^n \text{sen}[\omega n] u[n]$	$\frac{1-az^{-1} \cos \omega}{1-2az^{-1} \cos \omega + a^2 z^{-2}}$	$ z > a $

Transformada Z

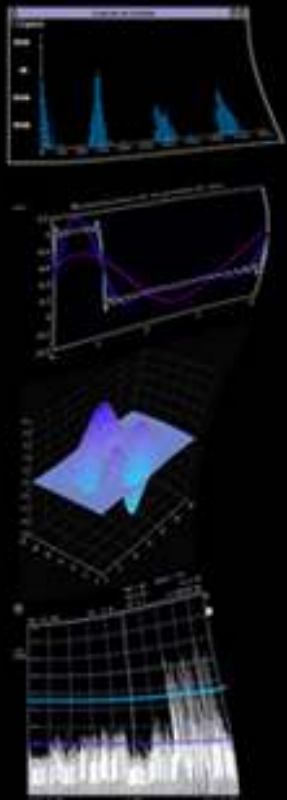
Propiedades

Linealidad

$$c_1x_1[n] + c_2x_2[n] \xleftrightarrow{TZ} c_1X_1(z) + c_2X_2(z) \quad ; \quad ROC = ROC_1 \cap ROC_2$$

Convolución

$$x_1[n] * x_2[n] \xleftrightarrow{TZ} X_1(z)X_2(z) \quad ; \quad ROC = ROC_1 \cap ROC_2$$



Transformada Z

Propiedades

Desplazamiento Temporal

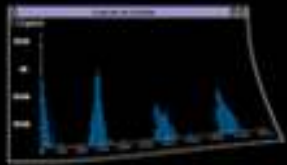
$$x[n-k] \stackrel{TZ}{\leftrightarrow} z^{-k} X(z) \quad ; \quad = ROC \vee ROC \not\subset 0 \vee ROC \not\subset \infty$$

Reflexión Temporal

$$x[-n] \stackrel{TZ}{\leftrightarrow} X(z^{-1}) \quad ; \quad ROC: \frac{1}{r_{\text{sup}}} < |z| < \frac{1}{r_{\text{inf}}}$$

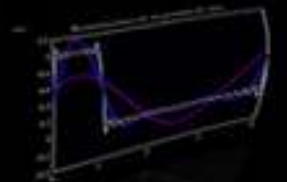
Transformada Z

Propiedades



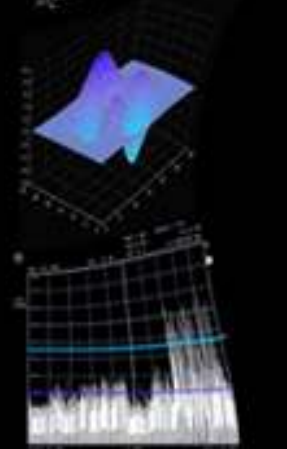
Escalado en el Dominio Z

$$a^n x[n] \stackrel{TZ}{\leftrightarrow} X\left(\frac{z}{a}\right) ; \text{ROC} : |a| r_{\text{inf}} < |z| < |a| r_{\text{sup}}$$



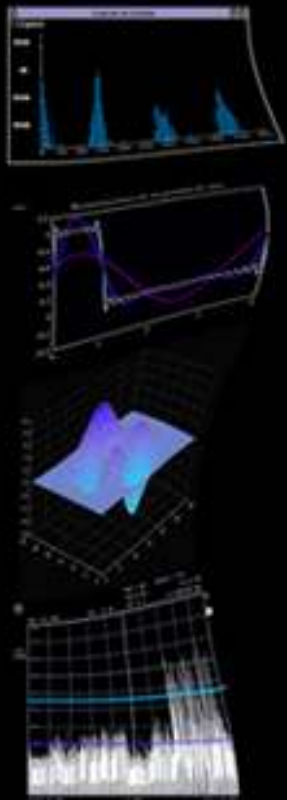
Diferenciación en el Dominio Z

$$nx[n] \stackrel{TZ}{\leftrightarrow} -z \frac{\partial X(z)}{\partial z} ; \text{ROC} \vee \text{ROC} \not\subset 0 \vee \text{ROC} \not\subset \infty$$



Transformada Z

Serie Geométrica



$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} = 1 + a + a^2 + \dots \quad \text{si } |a| < 1$$

$$\sum_{n=1}^{\infty} a^n = a + a^2 + \dots = \frac{1}{1-a} - 1 = \frac{a}{1-a} \quad \text{si } |a| < 1$$

$$\sum_{n=2}^{\infty} a^n = \frac{a^2}{1-a} \quad \text{si } |a| < 1$$

⋮

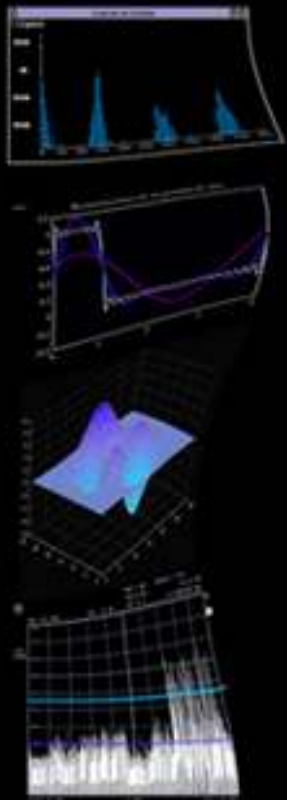
$$\sum_{n=N}^{\infty} a^n = \frac{a^N}{1-a} \quad \text{si } |a| < 1$$

⋮

$$\sum_{n=0}^{N-1} a^n = \sum_{n=0}^{\infty} a^n - \sum_{n=N}^{\infty} a^n = \frac{1-a^N}{1-a} \quad \text{si } |a| < 1$$

Transformada Z

Antitransformada Z

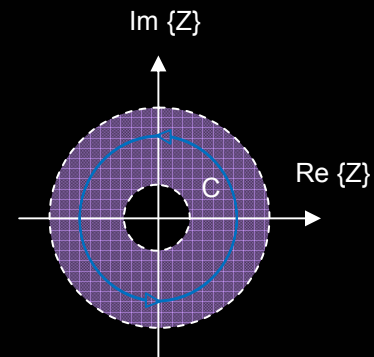


Cálculo directo

Integración en contorno C en sentido antihorario

$\text{ROC} \subset C$ y $z=0$

$\text{ROC} \subset z_i$ simples



Transformada Z

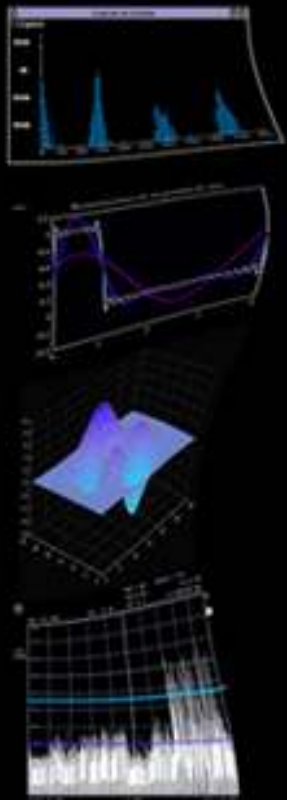
Antitransformada Z

Expansión en Serie de Potencias

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}} = \frac{b_M z^{-M} + \dots + b_1 z^{-1} + b_0}{a_N z^{-N} + \dots + a_1 z^{-1} + a_0}$$

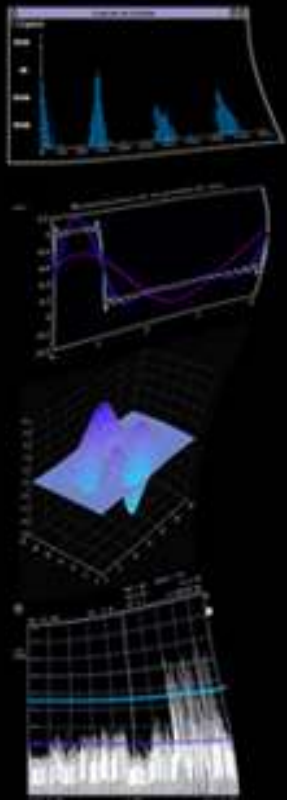
División de polinomios en orden creciente o decreciente

Por inspección se determina la antitransformada



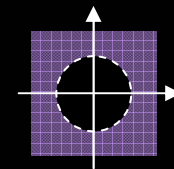
Transformada Z

Antitransformada Z



Expansión en Series de z^{-k}

si ROC: $|p_k| < |z|$

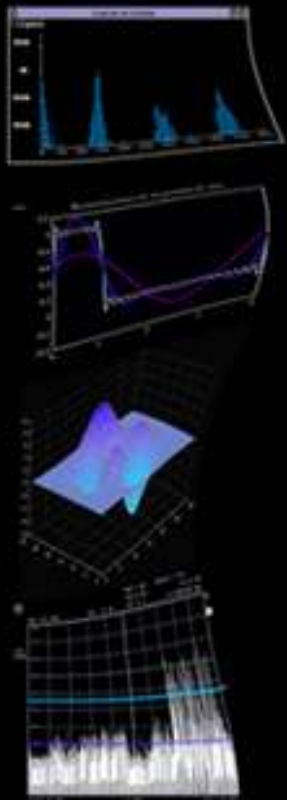


División de polinomios en orden decreciente

$$\begin{array}{r}
 b_0 + b_1 z^{-1} + \dots + b_M z^{-M} \\
 - \\
 b_0 + d_1 z^{-1} + \dots + d_M z^{-M} + \dots + d_N z^{-N} \\
 \hline
 e_1 z^{-1} + \dots + e_M z^{-M} + \dots + e_N z^{-N} \\
 - \\
 e_1 z^{-1} + \dots + f_M z^{-M} + \dots + f_N z^{-N} + \dots \\
 \hline
 \vdots
 \end{array}
 \quad
 \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}{c_0 + c_1 z^{-1} + \dots}$$

Transformada Z

Antitransformada Z

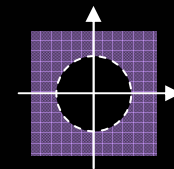


Expansión en Series de z^{-k}

Por inspección se determina la antitransformada

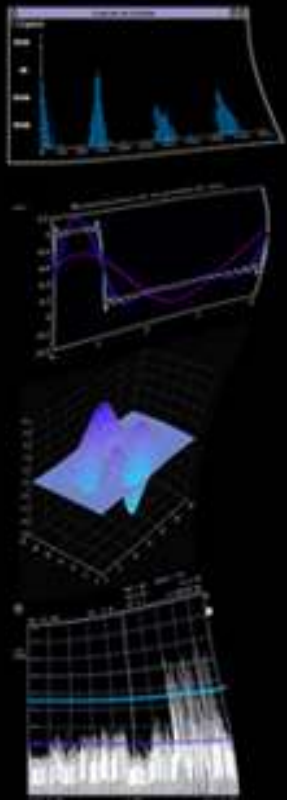
$$H(z) = \frac{b_0 + b_1z^{-1} + \dots + b_Mz^{-M}}{a_0 + a_1z^{-1} + \dots + a_Nz^{-N}} = c_0 + c_1z^{-1} + c_2z^{-2} + \dots$$

$$H(z) = c_0 + c_1z^{-1} + \dots \stackrel{z}{\leftrightarrow} \left\{ \underset{\uparrow}{c_0}; c_1; c_2; \dots; c_k; \dots \right\} = h[n]$$



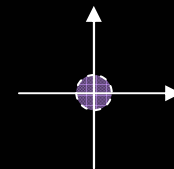
Transformada Z

Antitransformada Z



Expansión en Series de z^{+k}

si ROC: $|z| < |p_k|$

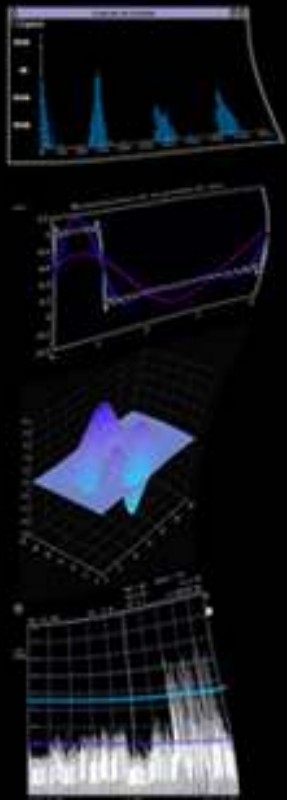


División de polinomios en orden creciente

$$\begin{array}{r}
 b_M z^{-M} + \dots + b_0 \\
 - \\
 b_M z^{-M} + \dots + d_0 + \dots + d_{-M+N} z^{-M+N} \\
 \hline
 + \dots + e_0 + \dots + e_{-M+N} z^{-M+N} \\
 - \\
 + \dots + f_0 + \dots + f_{-M+N} z^{-M+N} + \dots \\
 \hline
 \vdots
 \end{array}
 \quad
 \frac{a_N z^{-N} + \dots + a_2 z^{-2} + a_1 z^{-1} + a_0}{c_{M-N} z^{N-M} + c_{M+1-N} z^{N-M+1} + \dots}$$

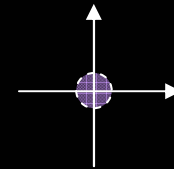
Transformada Z

Antitransformada Z



Expansión en Series de z^{+k}

Por inspección se determina la antitransformada



$$H(z) = \frac{b_M z^{-M} + \dots + b_1 z^{-1} + b_0}{a_N z^{-N} + \dots + a_1 z^{-1} + a_0} = c_{M-N} z^{N-M} + c_{M+1-N} z^{N-M+1} + \dots + c_{M+1-N} + \dots$$

$$H(z) = c_{M-N} z^{N-M} + c_{M+1-N} z^{N-M+1} + \dots \xleftrightarrow{z} \left\{ \dots; c_{M+1-N}; c_{M-N}; \dots; 0 \right\} = h[n]$$

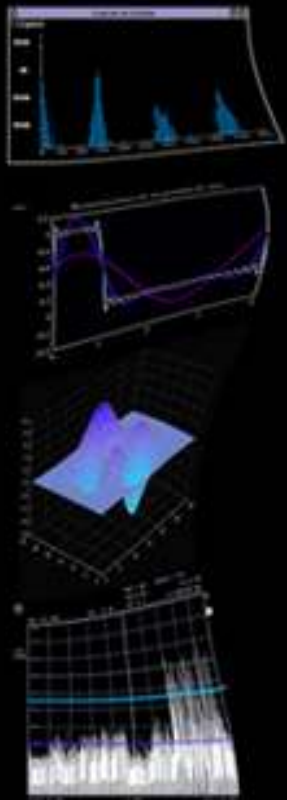
Transformada Z

Antitransformada Z

Expansión en Serie de Potencias

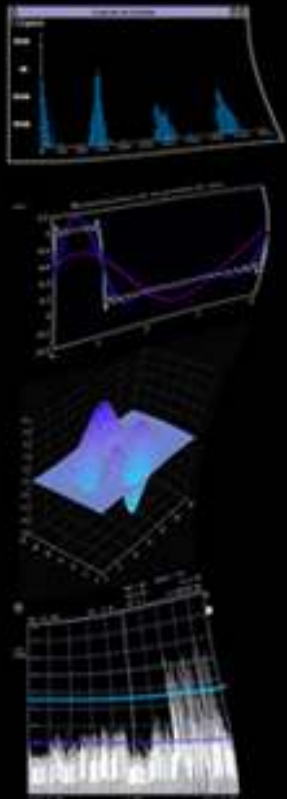
Una función $H(v)$ puede expresarse como serie de potencias

$$H(v) = \sum_{n=0}^{\infty} \frac{H^{(n)}(v_0)(v - v_0)^n}{n!}$$



Transformada Z

Antitransformada Z



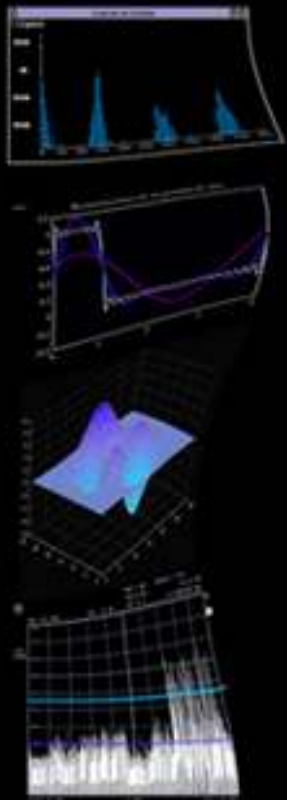
Expansión en Serie de Potencias

Una función $H(v)$ puede expresarse como serie de potencias

$$H(v) = \sum_{n=0}^{\infty} \frac{H^{(n)}(v_0)(v - v_0)^n}{n!}$$
$$= H(v_0) + H'(v_0)(v - v_0) + \frac{H''(v_0)(v - v_0)^2}{2} + \dots + \frac{H^{(k)}(v_0)(v - v_0)^k}{k!} + \dots$$

Transformada Z

Antitransformada Z



Expansión en Serie de Potencias

Una función $H(v)$ puede expresarse como serie de potencias

$$H(v) = \sum_{n=0}^{\infty} \frac{H^{(n)}(v_0)(v - v_0)^n}{n!}$$
$$= H(v_0) + H'(v_0)(v - v_0) + \frac{H''(v_0)(v - v_0)^2}{2} + \dots + \frac{H^{(k)}(v_0)(v - v_0)^k}{k!} + \dots$$

Para eso se calculan las derivadas y se evalúan en $v=v_0$

$$H(v) \Rightarrow H(v_0)$$

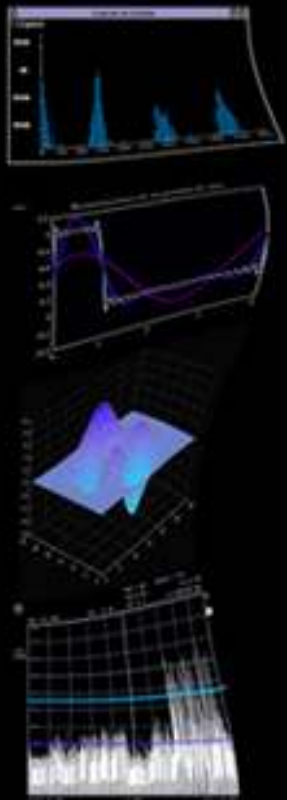
$$H'(v) \Rightarrow H'(v_0)$$

$$H''(v) \Rightarrow H''(v_0)$$

⋮

Transformada Z

Antitransformada Z



Serie de Potencias

Si $v_0=0$

$$H(v) = \sum_{n=0}^{\infty} \frac{H^{(n)}(0)(v)^n}{n!} = H(0) + H'(0)v + \frac{H''(0)v^2}{2} + \dots + \frac{H^{(k)}(0)v^k}{k!} + \dots$$

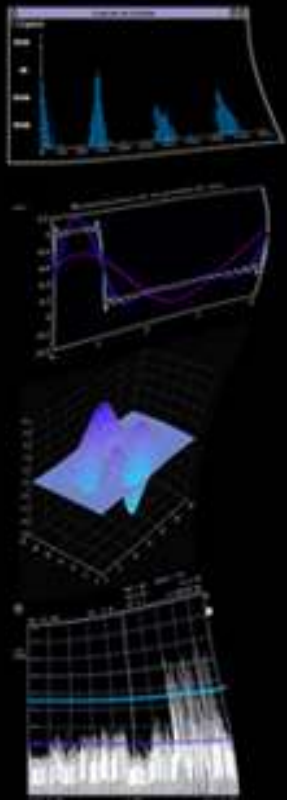
Si $v=z^{-1}$

$$H(z) = \sum_{n=0}^{\infty} \frac{H^{(n)}(0)z^{-n}}{n!} = H(0) + H'(0)z^{-1} + \frac{H''(0)z^{-2}}{2} + \dots + \frac{H^{(k)}(0)z^{-k}}{k!} + \dots$$

$$H(z) = \sum_{n=0}^{\infty} \frac{H^{(n)}(0)z^{-n}}{n!} \stackrel{z}{\leftrightarrow} \left\{ \underset{\uparrow}{H(0)}; H'(0); \frac{H''(0)}{2}; \dots; \frac{H^{(k)}(0)}{k!}; \dots \right\} = h[n]$$

Transformada Z

Antitransformada Z



Expansión en fracciones simples

Grados de los polinomios: $^{\circ}N(z)=M$ y $^{\circ}D(z)=N$

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}}$$

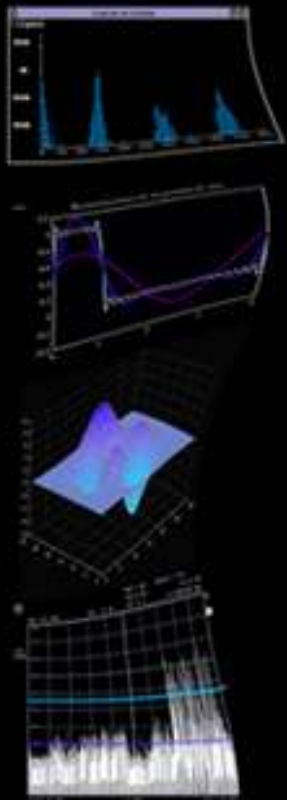
Si $M \geq N \rightarrow$ división previa para reducir el grado

Si $M < N$ y no $\exists z_i$ de orden $s \neq 1 \rightarrow$ cálculo directo

Si $M < N$ y $\exists z_i$ de orden $s \neq 1 \rightarrow$ cálculo separado

Transformada Z

Antitransformada Z



Expansión en fracciones simples

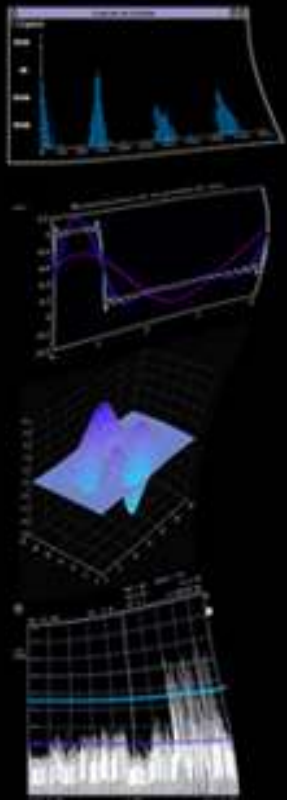
Si $M \geq N \rightarrow$ división previa para reducir el grado

$$H(z) = \frac{b_0 + b_1z^{-1} + \dots + b_Mz^{-M}}{a_0 + a_1z^{-1} + \dots + a_Nz^{-N}} = \frac{N(z)}{D(z)} = c_0 + c_1z^{-1} + \dots + c_{M-N}z^{-M-N} + \frac{R(z)}{D(z)}$$

donde $\text{°}R(z) < \text{°}D(z)$

Transformada Z

Antitransformada Z



Expansión en fracciones simples

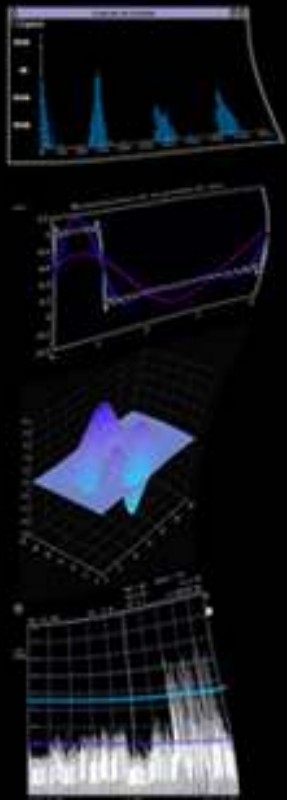
Si $M < N$ y no $\exists z_i$ de orden $s \neq 1 \rightarrow$ cálculo directo

$$H(z) = \sum_{k=1}^M \frac{A_k}{1 - p_k z^{-1}}$$

$$\frac{A_k}{1 - p_k z^{-1}} \xleftrightarrow{TZ} \begin{cases} A_k [p_k]^n u[n] & \text{si } ROC: |p_k| < |z| \\ -A_k [p_k]^n u[-n-1] & \text{si } ROC: |z| < |p_k| \end{cases}$$

Transformada Z

Antitransformada Z



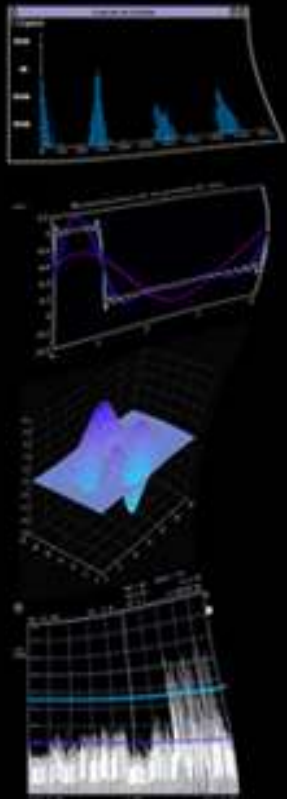
Expansión en fracciones simples

Polos complejos conjugados \leftrightarrow residuos complejos conjugados

$$\frac{A_k}{1 - p_k z^{-1}} + \frac{A_k^*}{1 - p_k^* z^{-1}} = \frac{b_0 + b_1 z^{-1}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

Transformada Z

Transformada Z unilateral



Transformada

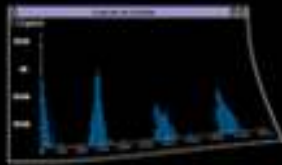
$$\mathcal{TZ}^+ \{x[n]\} = X^+(z) = \sum_{n=0}^{+\infty} x[n]z^{-n}$$

Región de convergencia

$$ROC : a < |z|$$

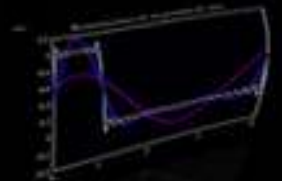
Transformada Z

Propiedades



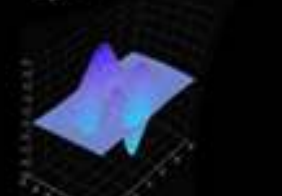
Retardo temporal

$$x[n-k] \stackrel{TZ^+}{\leftrightarrow} z^{-k} \left[X^+(z) + \sum_{n=1}^k x[-n]z^n \right] ; k > 0$$



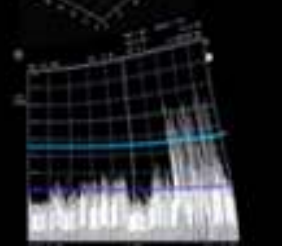
Avance Temporal

$$x[n+k] \stackrel{TZ^+}{\leftrightarrow} z^k \left[X^+(z) - \sum_{n=0}^{k-1} x[n]z^{-n} \right] ; k > 0$$



Teorema del Valor Final

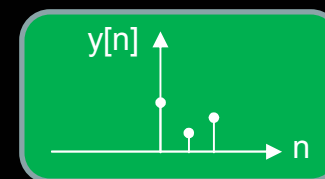
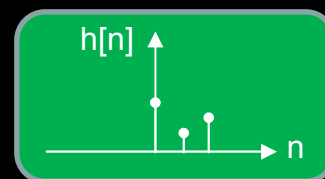
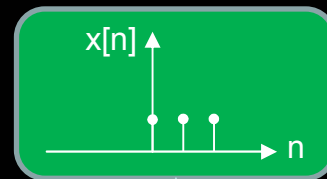
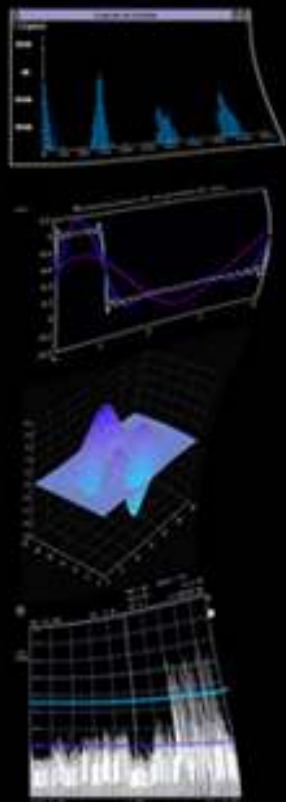
$$\lim_{n \rightarrow \infty} x[n] = \lim_{z \rightarrow 1} (z-1) X^+(z)$$



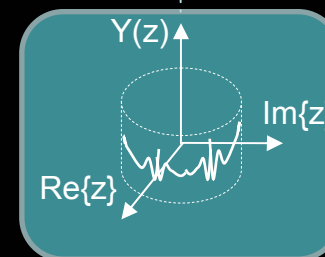
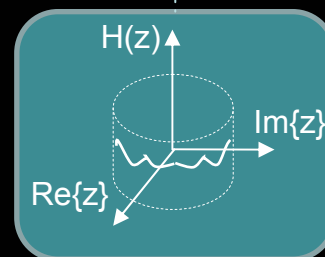
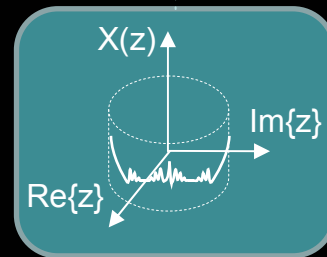
Transformada Z

Convolución

Transferencia $H(z)$

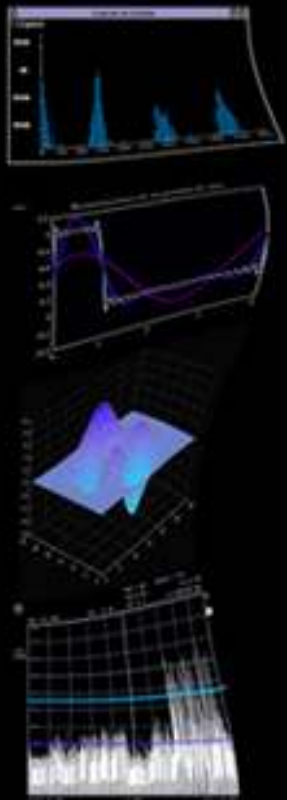


entrada \rightarrow $\mathcal{S}\{\cdot\}$ \rightarrow salida



Transformada Z

Convolución



$$y[n] = h[n] * x[n]$$

$$\updownarrow_{TZ} \qquad \qquad \updownarrow_{TZ}$$

$$Y(z) = H(z)X(z)$$

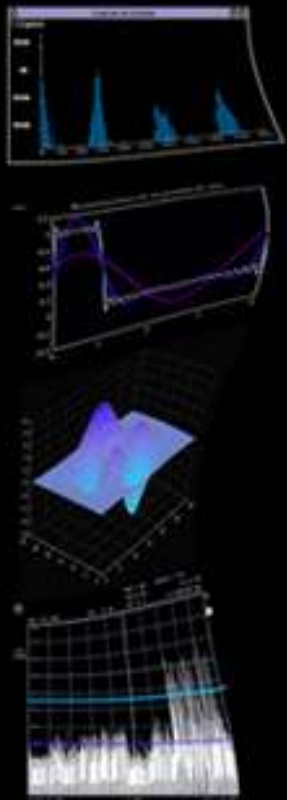
\Downarrow

$$H(z) = \frac{Y(z)}{X(z)}$$

Transformada Z

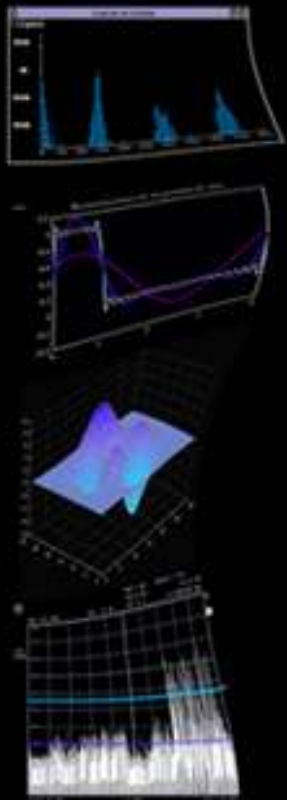
Ecuación en diferencias

$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$



Transformada Z

Ecuación en diferencias



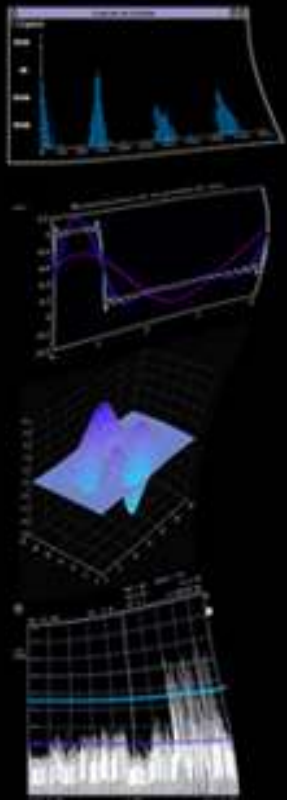
$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$

\Downarrow TZ \Downarrow TZ

$$\sum_{k=0}^N a_k Y(z)z^{-k} = \sum_{k=0}^M b_k X(z)z^{-k}$$

Transformada Z

Ecuación en diferencias



$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$

\Downarrow TZ

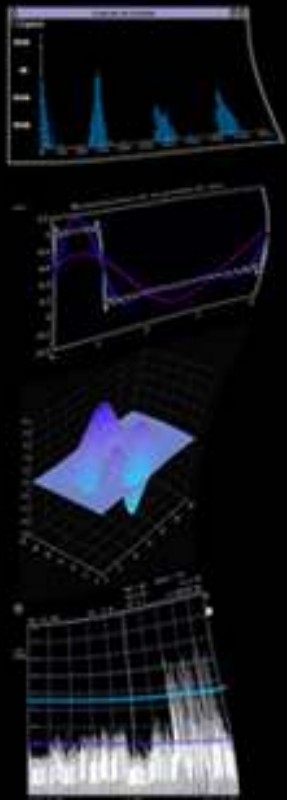
\Downarrow TZ

$$\sum_{k=0}^N a_k Y(z) z^{-k} = \sum_{k=0}^M b_k X(z) z^{-k}$$

$$Y(z) \sum_{k=0}^N a_k z^{-k} = X(z) \sum_{k=0}^M b_k z^{-k}$$

Transformada Z

Ecuación en diferencias



$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$

\updownarrow TZ \updownarrow TZ

$$\sum_{k=0}^N a_k Y(z) z^{-k} = \sum_{k=0}^M b_k X(z) z^{-k}$$

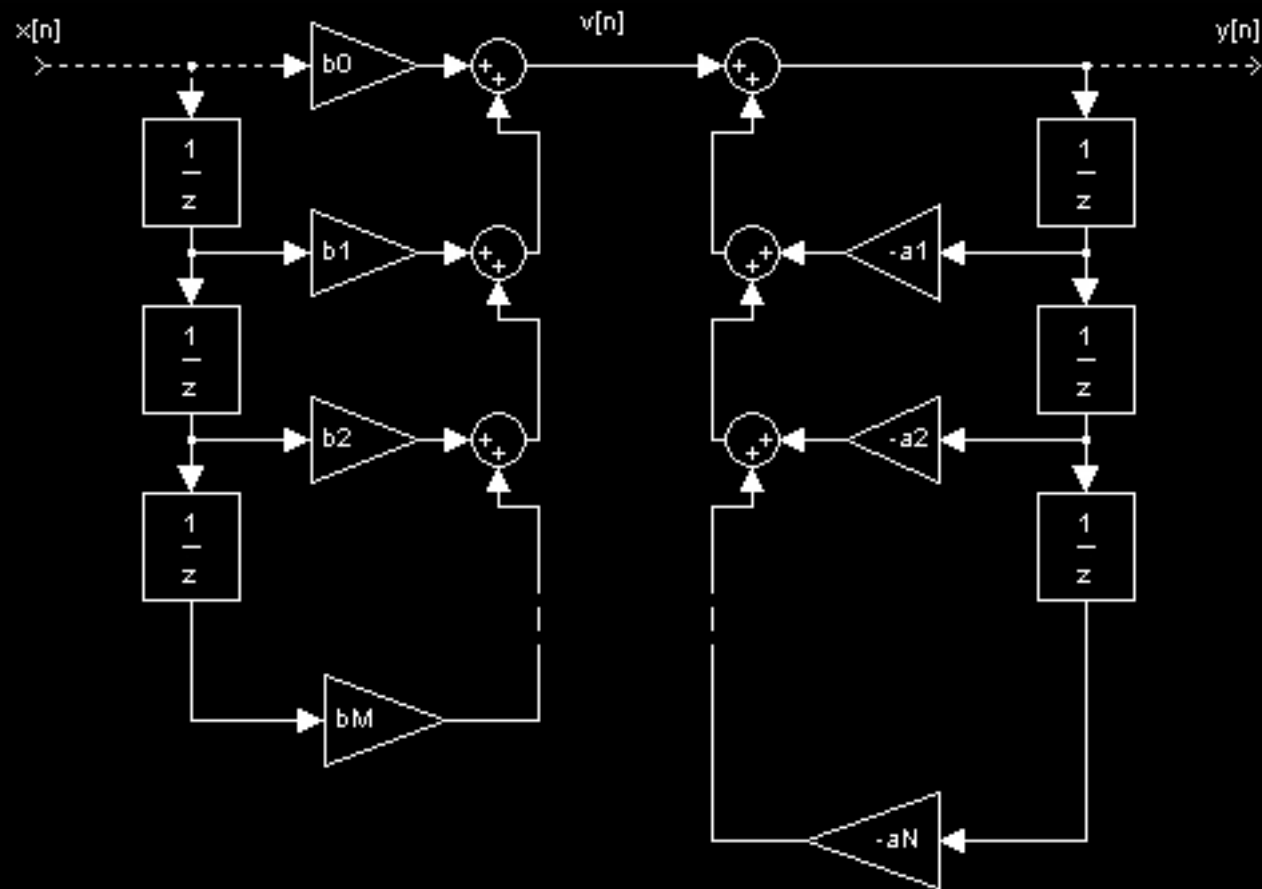
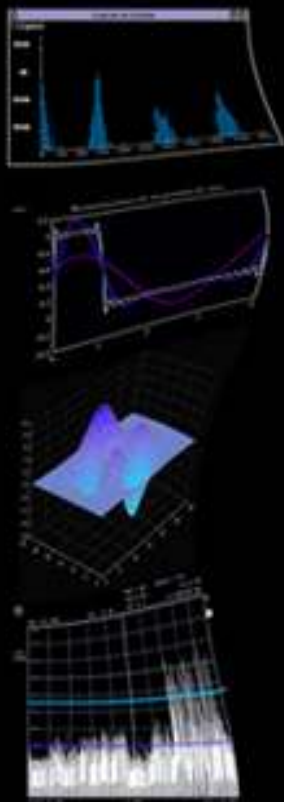
$$Y(z) \sum_{k=0}^N a_k z^{-k} = X(z) \sum_{k=0}^M b_k z^{-k}$$

\Downarrow

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}}$$

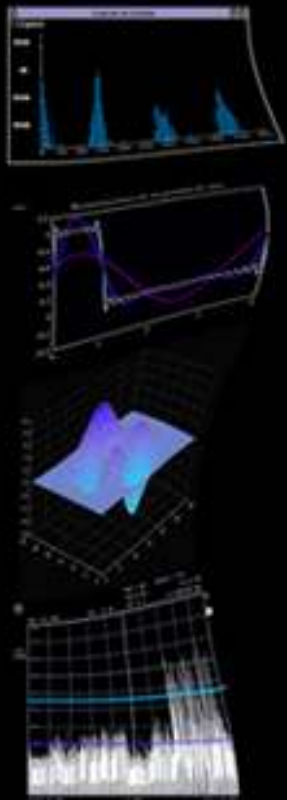
Transformada Z

Ecuación en diferencias



Transformada Z

Análisis de sistemas



$$y[n] = x[n] * h[n]$$



$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$

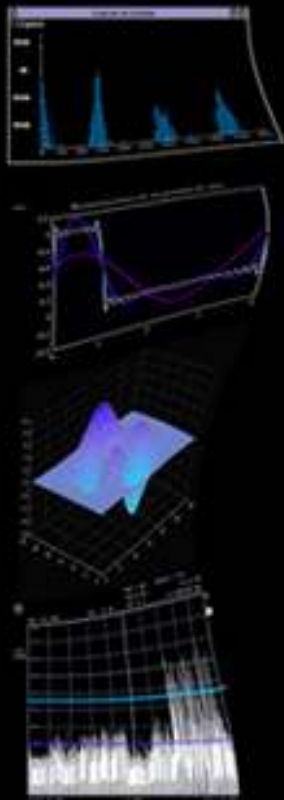


$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}}$$



Transformada Z

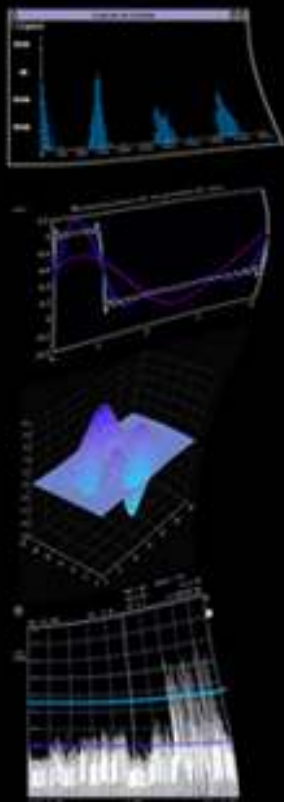
Análisis de sistemas LIT



FIR	IIR
<p>MA (Promediador Móvil)</p> $H(z) = \sum_{k=0}^M b_k z^{-k} = \frac{1}{z^M} \sum_{k=0}^M b_k z^{M-k}$ <p>M ceros y 1 polo de orden M</p>	<p>AR (Autoregresivo)</p> $H(z) = \frac{b_0}{\sum_{k=0}^N a_k z^{-k}} = \frac{b_0 z^N}{\sum_{k=0}^N a_k z^{N-k}}$ <p>N polos y 1 cero de orden N</p>
	<p>ARMA (Promediador Móvil Autoregresivo)</p> $H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}}$ <p>M ceros y N polos no triviales</p>

Transformada Z

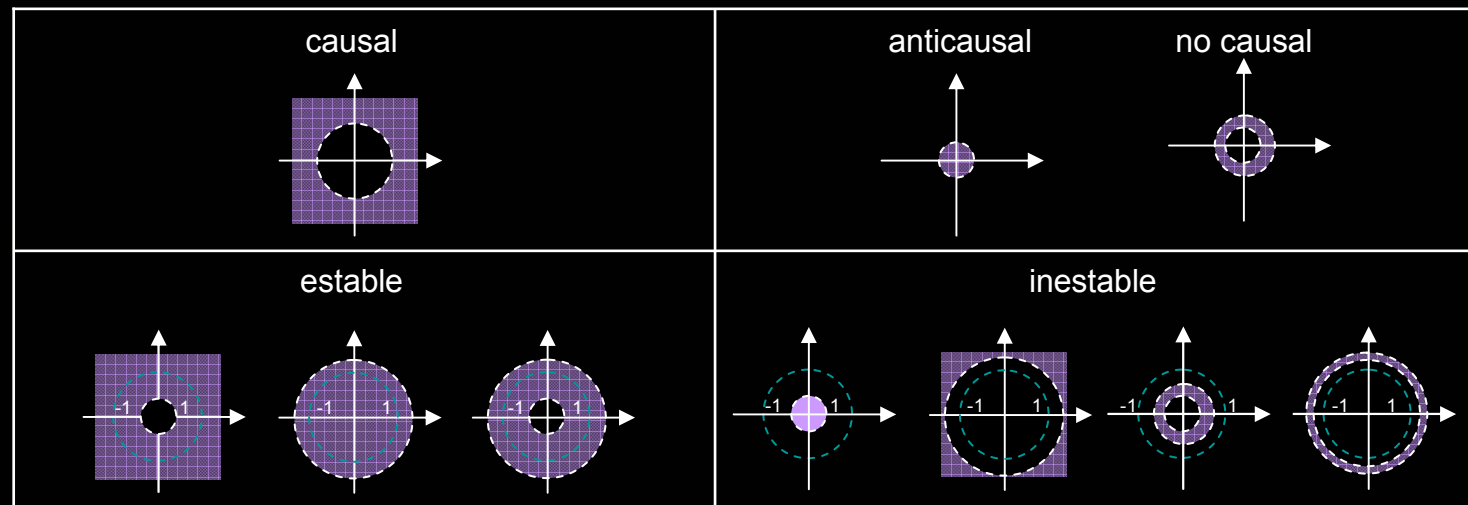
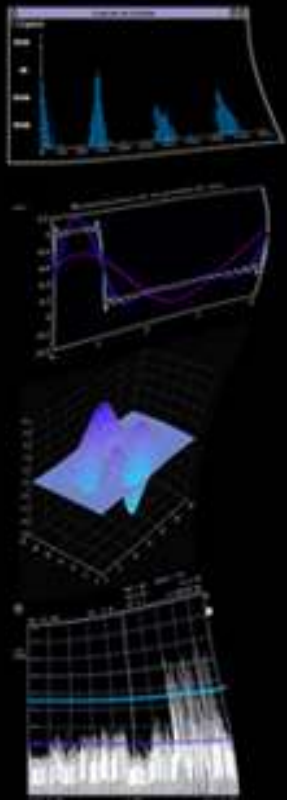
ROC



Lateralidad	Longitud infinita	Longitud finita
Derecha		
Bilateral		
Izquierda		

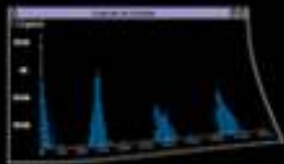
Transformada Z

Análisis de sistemas LIT



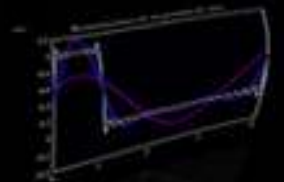
Transformada Z

Análisis de sistemas LIT



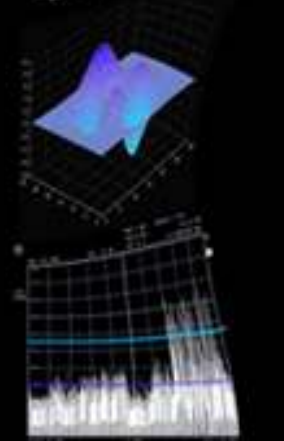
Estabilidad

$ROC : 1 < |z| \Rightarrow H(z)$ es SE



Causalidad

$ROC : a < |z| \Rightarrow H(z)$ es SC

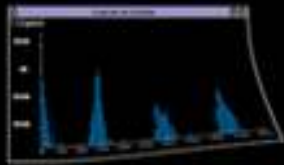


Respuesta espectral

$$ROC \subset |z| = 1 \Rightarrow H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}} = \frac{\prod_{s=1}^M (z - c_s)}{\prod_{k=1}^N (z - p_k)}$$

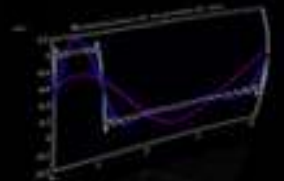
Transformada Z

Análisis de sistemas LIT



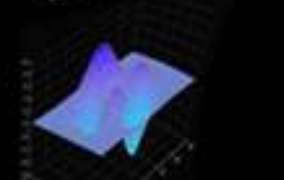
Transferencia

$$H(z) = \frac{B(z)}{A(z)} \text{ con polos } p_k \quad k = 0; 1; \dots; N$$



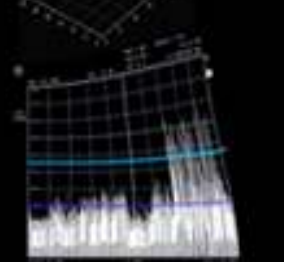
Entrada

$$X(z) = \frac{Q(z)}{N(z)} \text{ con polos } q_k \neq p_k \quad k = 0; 1; \dots; L$$



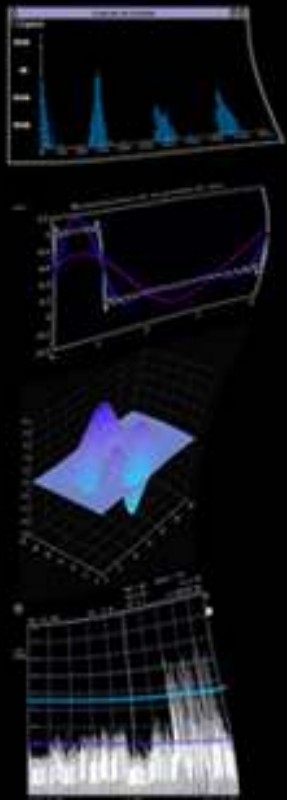
Salida

$$Y(z) = X(z)H(z) = \frac{Q(z) B(z)}{N(z) A(z)}$$



Transformada Z

Análisis de sistemas LIT



Salida

$$Y(z) = X(z)H(z) = \frac{B(z)Q(z)}{A(z)N(z)} = \sum_{k=1}^N \frac{A_k}{1-p_k z^{-1}} + \sum_{k=1}^L \frac{Q_k}{1-q_k z^{-1}}$$

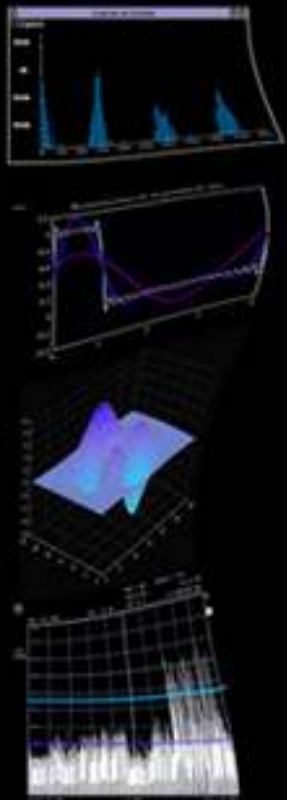
$$y[n] = \sum_{k=1}^N A_k p_k^n u[n] + \sum_{k=1}^L Q_k q_k^n u[n]$$

↑
Respuesta
Natural

↑
Respuesta
forzada

Transformada Z

Análisis de sistemas LIT



Salida

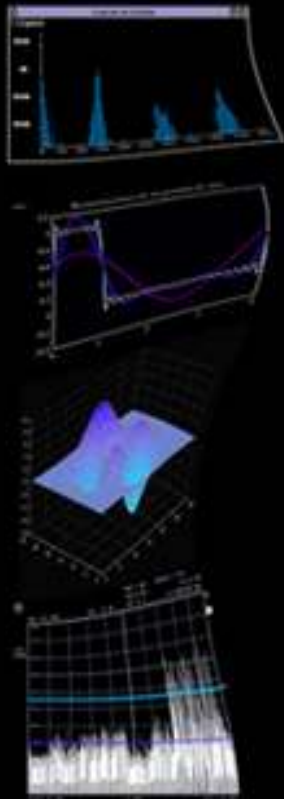
$$Y(z) = \sum_{k=0}^M b_k X(z) z^{-k} - \sum_{k=1}^N a_k Y(z) z^{-k}$$

$$Y^+(z) = \sum_{k=0}^M b_k X^+(z) z^{-k} - \sum_{k=1}^N a_k \left[Y^+(z) + \sum_{n=1}^k y[-n] z^n \right] z^{-k}$$

$$Y^+(z) = \frac{\sum_{k=0}^M b_k X(z) z^{-k} - \sum_{k=1}^N a_k z^{-k} \sum_{n=1}^k y[-n] z^n}{1 + \sum_{k=1}^N a_k z^{-k}} = \frac{B(z)}{A(z)} X(z) \frac{N_0(z)}{A(z)}$$

Transformada Z

Análisis de sistemas LIT



Salida

$$Y^+(z) = \sum_{k=1}^N \frac{A_k + D_k}{1 - p_k z^{-1}} + \sum_{k=1}^L \frac{Q_k}{1 - q_k z^{-1}}$$

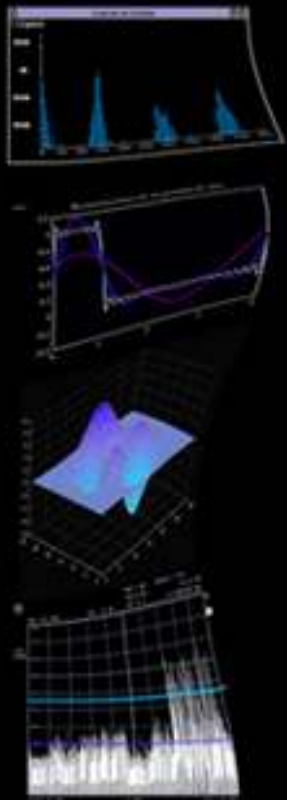
$$y[n] = \sum_{k=1}^N (A_k + D_k) p_k^n u[n] + \sum_{k=1}^L Q_k q_k^n u[n]$$

↑
Respuesta
natural
(ZS + ZI)

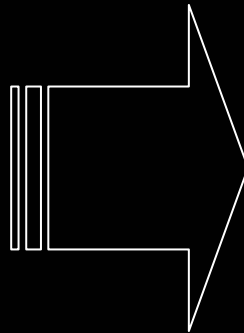
↑
Respuesta
forzada

Transformada Z

Análisis de sistemas LIT



$$H(z) = \frac{\prod_{s=1}^M (z - c_s)}{\prod_{k=1}^N (z - p_k)}$$



Módulo

Es la relación entre los productos de las distancias del círculo unitario $e^{j\omega}$ a cada cero y a cada polo

$$|H(e^{j\omega})| = \frac{\prod_{s=1}^M |e^{j\omega} - c_s|}{\prod_{k=1}^N |e^{j\omega} - p_k|}$$

Fase

Es la resultante de las fases de los vectores que van de los ceros y polos al número complejo $e^{j\omega}$

$$\angle H(e^{j\omega}) = \sum_{s=1}^M \angle (e^{j\omega} - c_s) - \sum_{k=1}^N \angle (e^{j\omega} - p_k)$$

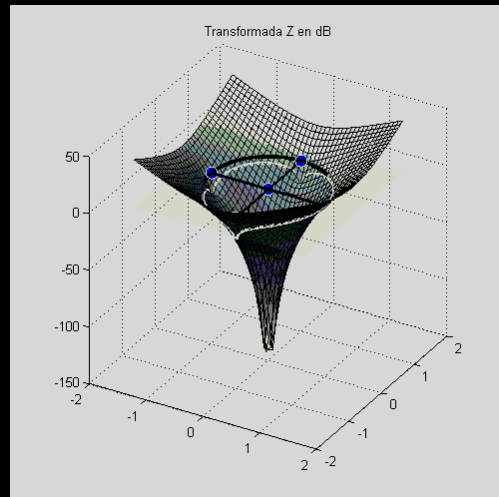
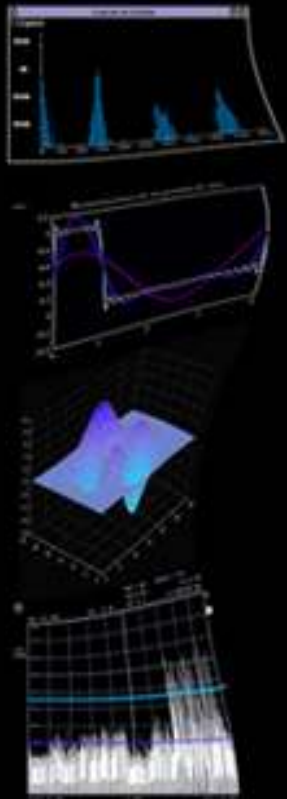
Transformada Z

zt

Computa y grafica la Transformada Z en dB de una transferencia

Sintaxis

`[Hz, Hw, z, w, c, p]=zt(b, a, f, graficar)`



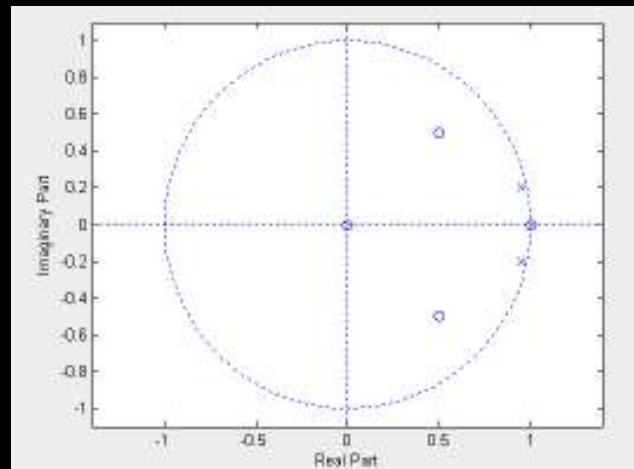
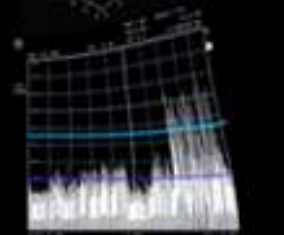
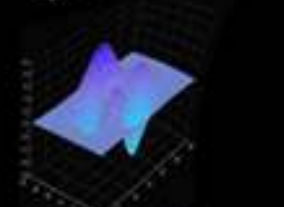
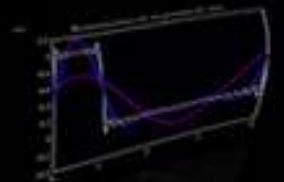
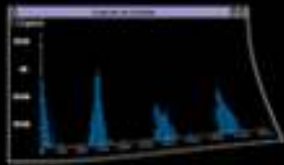
Transformada Z

zplane

Grafica el mapa de polos y ceros en el plano Z

Sintaxis

`zplane [z, p]`



Transformada Z

impz

Computa la respuesta al impulso de un sistema discreto

Sintaxis

```
[h,t] = impz(b,a,n,fs)  
impz(b,a,n,fs)
```

